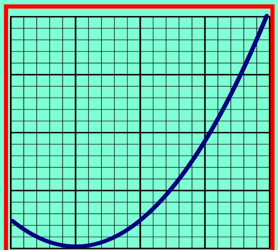
Math 125

Fall 2021

Lecture 30



$y = ax^2 + bx + c$

Class QZ 24

Solve by matrix Method:

$$\begin{cases} 3x - 5y = 7 \\ x - y = 1 \end{cases}$$

Augmented Matrix

$$\left[\begin{array}{cc|c} 3 & -5 & 7 \\ 1 & -1 & 1 \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 3 & -5 & 7 \end{array}\right]$$

$(-3)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & -2 & 4 \end{array}\right]$$

$R2 \div (-2) \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 1 & -2 \end{array}\right]$$

$R2 + R1 \rightarrow R1$

$$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -2 \end{array}\right] \begin{array}{l} x = -1 \\ y = -2 \end{array} \Rightarrow \boxed{(-1, -2)}$$

$\{(-1, -2)\}$

October 25, 2021

Solve by matrix Method:

$$\begin{cases} x - 3y + z = 2 \\ 4x - 12y + 4z = 8 \\ -2x + 6y - 2z = -4 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 1 & 2 \\ 4 & -12 & 4 & 8 \\ -2 & 6 & -2 & -4 \end{array}\right]$$

$(-4)R1 + R2 \rightarrow R2$

$(2)R1 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Row made of all Zeros $\Rightarrow$ infinite # of Solutions

Solve by matrix Method:

$$\begin{cases} x \quad\quad -z = -2 \\ \quad y + 3z = 11 \\ x + y + z = 6 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 3 & 11 \\ 1 & 1 & 1 & 6 \end{array}\right]$$

$(-1)R1 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 3 & 11 \\ 0 & 1 & 2 & 8 \end{array}\right]$$

$(-1)R2 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 3 & 11 \\ 0 & 0 & -1 & -3 \end{array}\right]$$

$R3 \div (-1) \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 3 & 11 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$(-3)R3 + R2 \rightarrow R2$

$R3 + R1 \rightarrow R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}$$

$\{(1, 2, 3)\}$

Rational Exponent & Radical notations

$$x^{\frac{m}{n}} \rightarrow \sqrt[n]{x^m}$$

Index

Radical

Radicand

$$x^{\frac{2}{3}} = \sqrt[3]{x^2}$$

$$x^{\frac{3}{4}} = \sqrt[4]{x^3}$$ Index = 4, Radicand = x^3

No index → index = 2 → Square-Root.

$$x^{1/2} = x^{\frac{1}{2}} = \sqrt[2]{x^1} = \sqrt{x}$$

$\sqrt[3]{x^5}$ Index = 3, Radicand = x^5

$\sqrt[7]{x^2}$ Index = 7, Radicand = x^2

$\sqrt{x^9}$ No index, index = 2, Radicand = x^9

When index is even:

Radicand & Answer ⇒ Non-negative.

When index is odd:

Radicand & Answer ⇒ Both must have Same Sign

$$\sqrt[n]{x} = y \iff y^n = x$$

Answer$^{\text{index}}$ = Radicand

$$\sqrt[4]{x} = y \iff y^4 = x$$

$y \geq 0$, $x \geq 0$

$$\sqrt[5]{x} = y \iff y^5 = x$$

x & y have same sign

both + or both −

Some operations

$$\sqrt[2]{x^1} \cdot \sqrt[3]{x^1} = x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}}$$

$$= x^{\frac{5}{6}}$$

$$= \sqrt[6]{x^5}$$

Exponential Rules: $\boxed{x^m \cdot x^n = x^{m+n}}$

$$\frac{1\cdot 3}{2\cdot 3} + \frac{1\cdot 2}{3\cdot 2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Index = 6

Radicand = x^5

Simplify: $\sqrt[3]{x^2} \cdot \sqrt[4]{x^1}$

$= x^{\frac{2}{3}} \cdot x^{\frac{1}{4}}$

$= x^{\frac{2}{3}+\frac{1}{4}}$

$= x^{\frac{11}{12}} = \boxed{\sqrt[12]{x^{11}}}$

$\frac{2\cdot4}{3\cdot4} + \frac{1\cdot3}{4\cdot3}$

$= \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

Simplify $\frac{\sqrt[5]{x^3}}{\sqrt[2]{x^1}} = \frac{x^{\frac{3}{5}}}{x^{\frac{1}{2}}} = x^{\frac{3}{5}-\frac{1}{2}}$

$= x^{\frac{1}{10}}$

$= \sqrt[10]{x^1}$

$= \boxed{\sqrt[10]{x}}$

Exponential Rule

$\frac{x^m}{x^n} = x^{m-n}$

$\frac{3\cdot2}{5\cdot2} - \frac{1\cdot5}{2\cdot5} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$

Simplify: $\dfrac{\sqrt[5]{x^3}\cdot\sqrt[3]{x}}{\sqrt[4]{x^3}}$

$$= \frac{x^{\frac{3}{5}}\cdot x^{\frac{1}{3}}}{x^{\frac{3}{4}}} = x^{\frac{3}{5}+\frac{1}{3}-\frac{3}{4}}$$

$$= x^{\frac{11}{60}} = \boxed{\sqrt[60]{x^{11}}}$$

$$\frac{3\cdot 12}{5\cdot 12} + \frac{1\cdot 20}{3\cdot 20} - \frac{3\cdot 15}{4\cdot 15}$$

LCD $= 5\cdot 3\cdot 4$
$= 60$

$$= \frac{36}{60} + \frac{20}{60} - \frac{45}{60} = \frac{36+20-45}{60} = \boxed{\frac{11}{60}}$$

Some Properties:

$$\sqrt[n]{A}\,\sqrt[n]{B} = \sqrt[n]{AB}$$

$$\frac{\sqrt[n]{A}}{\sqrt[n]{B}} = \sqrt[n]{\frac{A}{B}}$$

$$\sqrt[3]{10}\cdot\sqrt[3]{2} = \sqrt[3]{10\cdot 2}$$
$$= \sqrt[3]{20}$$

$$\frac{\sqrt[5]{15}}{\sqrt[5]{3}} = \sqrt[5]{\frac{15}{3}}$$
$$= \sqrt[5]{5}$$

Simplify

$$\sqrt[2]{6}\,\sqrt[2]{10} = \sqrt{6\cdot 10} = \sqrt{60}$$
$$= \sqrt{4\cdot 15}$$
$$= \sqrt{4}\sqrt{15}$$
$$= \boxed{2\sqrt{15}}$$

Simplify

$\sqrt{5}\ \sqrt{15} = \sqrt{5\cdot 15}$

$= \sqrt{75}$

$= \sqrt{25\cdot 3} = \sqrt{25}\sqrt{3}$

$= \boxed{5\sqrt{3}}$

Simplify

Hint: Distribute

$\sqrt{3}(\sqrt{6} + \sqrt{3})$

$= \sqrt{3}\sqrt{6} + \sqrt{3}\sqrt{3} = \sqrt{18} + \sqrt{9}$

$= \sqrt{9\cdot 2} + \sqrt{9}$

$= \sqrt{9}\sqrt{2} + \sqrt{9}$

$= \boxed{3\sqrt{2} + 3}$

Simplify

Hint: FOIL

$(\sqrt{2} + 1)(\sqrt{2} - 1)$

$= \sqrt{2}\sqrt{2} - \sqrt{2}\cdot 1 + 1\cdot\sqrt{2} - 1\cdot 1$

$= \sqrt{4} - \cancel{\sqrt{2}} + \cancel{\sqrt{2}} - 1 = 2 - 1 = \boxed{1}$

FOIL & Simplify

$(2\sqrt{3} + 4)(\sqrt{3} - 2) = 2\sqrt{9} - \cancel{4\sqrt{3}} + \cancel{4\sqrt{3}} - 8$

$= 2\cdot 3 - 8 = 6 - 8 = \boxed{-2}$

Solving Simple Radical Equation:

$\sqrt[2]{x-3} = 5$

square both sides

$(\sqrt{x-3})^2 = (5)^2$

$x-3 = 25$

$x = 28$

$\sqrt{28-3} \overset{?}{=} 5$

$\sqrt{25} \overset{?}{=} 5$

$5 = 5$ ✓

$\{28\}$

Solve

$\sqrt[3]{2x+1} - 5 = 0$

$\sqrt[3]{2x+1} = 5$

$(\sqrt[3]{2x+1})^3 = (5)^3$

$2x+1 = 125$

$2x = 124$

$x = 62$

$\sqrt[3]{2(62)+1} - 5 = 0$

$\sqrt[3]{125} - 5 = 0$

$5 - 5 = 0$ ✓

$\{62\}$

Solve $\sqrt{2x+6}+3=1$

Isolate the radical

$\sqrt{2x+6}=1-3$

No index

$\sqrt{2x+6}=-2$

Index = 2

$\left(\sqrt{2x+6}\right)^2=(-2)^2$

$2x+6=4$

$2x=4-6$

$2x=-2$

$x=-1$

Check

$\sqrt{2(-1)+6}=-2$

$\sqrt{-2+6}=-2$

$\sqrt{4}=-2$

$2=-2$

False

No Solution

$\emptyset$

extraneous Solution

Solve

$\sqrt[3]{2x-7}-\sqrt[3]{x+5}=0$

Isolate one radical

$\sqrt[3]{2x-7}=\sqrt[3]{x+5}$

$\left(\sqrt[3]{2x-7}\right)^3=\left(\sqrt[3]{x+5}\right)^3$

$2x-7=x+5 \Rightarrow \boxed{x=12}$ ✓ $\{12\}$

Class QZ 25

Evaluate:

$$\begin{vmatrix} 2 & 3 & -4 \\ 1 & -1 & 5 \\ 3 & 2 & 1 \end{vmatrix} = 2\begin{vmatrix} -1 & 5 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} + (-4)\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}$$

$$= 2(-1-10) - 3(1-15) - 4(2+3)$$

$$= -22 + 42 - 20 = \boxed{0}$$